Geometry
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Synopsis Euclid Book I
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**Construction 1. Equilateral Triangle**
Given a segment, it is possible to construct an equilateral triangle on that segment.

**Construction 2. Transference of Distance**
Given a segment and a point, it is possible to construct a segment congruent to the given segment at the given point.

**Construction 3. Cut-off**
Given a segment and a ray, it is possible to construct a segment congruent to the given segment on the ray at the endpoint of the ray.

**Proposition 4. Side-Angle-Side (SAS)**
Given two triangles, if there is a correspondence between the vertices such that two pairs of corresponding sides are congruent, and the pair of included angles are congruent, then the triangles are congruent.

**Proposition 5. Base Angle Theorem**
Given an isosceles triangle, the base angles are congruent.

**Proposition 6. Converse of Base Angle Theorem**
Given a triangle and side as base, if the base angles are congruent, then the triangle is isosceles.

**Proposition 8. Side-Side-Side (SSS)**
Given two triangles, if there is a correspondence between the vertices such that the three pairs of corresponding sides are congruent, then the triangles are congruent.

**Construction 9. Angle Bisection**
Given an angle, it is possible to construct a ray which bisects it.

**Construction 10. Segment Bisection**
Given a segment, it is possible to construct its midpoint.

**Construction 11. Perpendicular on Segment**
Given a segment and a point on it, it is possible to construct a line perpendicular to the segment and through the point.

**Construction 12. Perpendicular of Segment**
Given a line and a point not on it, it is possible to construct a line perpendicular to the given line and through the point.

**Proposition 13. Linear Pairs**
Given a segment and a segment set upon it, the resulting adjacent angles are supplementary.

**Proposition 14. Converse to Linear Pairs**
Given three rays with a common endpoint, if two adjacent angles are supplementary, then the outer rays form a line.

**Proposition 15. Vertical Pairs** If two lines intersect, the vertical angles are congruent.

**Construction 22.** Given three line segments with no one greater in length than the other two together, it is possible to construct a triangle whose sides are congruent to the given segments.

**Construction 23.** Given an angle and a ray, it is possible to construct on that ray an angle congruent to the given angle.

**Proposition 26. Part A: Angle-Side-Angle (ASA)**
Given two triangles, if there is a correspondence between the vertices such that two pairs of corresponding angles are congruent, and the pair of included sides are congruent, then the triangles are congruent.

**Proposition 26. Part B: Angle-Angle-Side (AAS)**
Given two triangles, if there is a correspondence between the vertices such that two pairs of corresponding angles are congruent, and a pair of corresponding nonincluded sides are congruent, then the triangles are congruent.

**Proposition 27. Alternate Congruence Implies Parallelness**
Given a transversal, if alternate angles are congruent, then the lines are parallel.

**Proposition 29. Part A: Corresponding Congruence Implies Parallelness**
Given a transversal, if corresponding angles are congruent, then the lines are parallel.

**Proposition 29. Part B: Consecutive Supplements Imply Parallelness**
Given a transversal, if consecutive angles are supplementary, then the lines are parallel.

**Proposition 30. Parallelness implies Angle Congruence**
Given a transversal, if the lines are parallel, then the alternate angles are congruent, the corresponding angles are congruent, and the consecutive angles are supplementary.

**Proposition 31.** Lines parallel to the same line are parallel to each other.

**Construction 31.** Given a line and a point not on it, it is possible to construct a line parallel to the given line and through the given point.

**Proposition 32. Part A: Exterior Angle Theorem** Given a triangle, the measure of an exterior angle equals the sum of the measures of the opposite interior angles.

**Proposition 32. Part B: Sum of Angles Theorem** Given a triangle, the sum of the measures of the interior angles equals measure of two right angles.

**Proposition 33. Parallelogram Theorem** Given a pair of segments, if they are parallel and congruent, then the segments connecting the endpoints are parallel and congruent.

Euclid's Propositions I.34 through I.41 can be summarized as follows.

**Fact 36. Area of a Parallelogram**
Given a parallelogram with base $b$ and height $h$, the area is $A = bh$.

**Fact 38. Area of a Triangle**
Given a triangle with base $b$ and height $h$, the area is $A = \frac{1}{2}bh$.

**Construction 46. Square**
Given a segment, it is possible to construct a square on that segment.

**Proposition 47. Pythagorean Theorem**
Given a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.

**Proposition 48. Converse of Pythagorean Theorem**
Given a triangle such that the square of one side equals the sum of the squares of the other two sides, the angle opposite the longer side is a right angle.